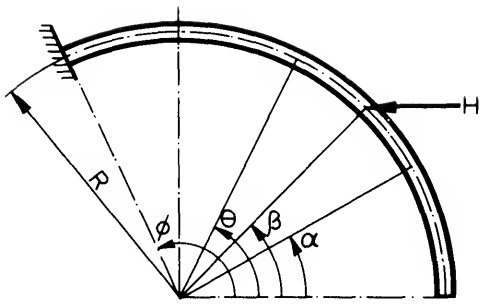

Fig. 25.2 General case of arched cantilever under horizontal load.

The corresponding deflection formulas for a general case of horizontal loading are then, when $\alpha < \beta$

$$Y = \frac{HR^3}{EI}[0.25(\cos 2\phi - \cos 2\beta) + \cos\alpha(\cos\beta - \cos\phi) - (\phi - \beta)\sin\beta\cos\alpha + \sin\phi\sin\beta - \sin^2\beta] \tag{25.9}$$

$$X = \frac{HR^3}{EI}[0.5(\phi - \beta) + 0.25(\sin 2\beta - \sin 2\phi) - (\cos\beta - \cos\phi)(\sin\alpha + \sin\beta) + (\phi - \beta)\sin\alpha\sin\beta] \tag{25.10}$$

When $\alpha > \beta$:

$$Y = \frac{HR^3}{EI}[0.25(\cos 2\phi - \cos 2\alpha) - (\phi - \alpha)\sin\beta\cos\alpha + \sin\beta(\sin\phi - \sin\alpha) + \cos^2\alpha - \cos\phi\cos\alpha] \tag{25.11}$$

$$X = \frac{HR^3}{EI}[0.5(\phi - \alpha) + 0.25(\sin 2\alpha - \sin 2\phi) + (\phi - \alpha)\sin\alpha\sin\beta - (\cos\alpha - \cos\phi)(\sin\alpha + \sin\beta)] \tag{25.12}$$

When $\alpha = \beta = 0$, Eqs. (25.9) to (25.12) yield simplified design formulas for the vertical and horizontal displacements, which can be expressed as follows:

$$Y = \frac{HR^3K_2}{EI} \tag{25.13}$$

and

$$X = \frac{HR^3K_3}{EI} \tag{25.14}$$

It is well to observe that the vertical displacement resulting from a horizontal load is numerically equal to the horizontal displacement due to a vertical load, provided that $P = H$. It is also of interest to recall that this correspondence of deflections agrees with the Maxwell's theorem of reciprocity for elastic structures [133, 134].